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PRESSURE AND ENERGY DENSITY DISTRIBUTION INSIDE A PROTON

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Abstract

In this work, a study on energy density and pressure distribution inside a proton has been performed and a trial has been made to establish energy density distribution function and pressure distribution function applicable inside the proton.

Keywords: *Gaussian Wave Packet, energy density distribution inside proton, pressure distribution inside proton.*

A set of Gaussian One Particle Wave Packet [1, 2] at initial time $t = 0$ is considered as a perfect fluid droplet which shows spherical symmetry. Total energy of a particle is distributed throughout the wave packet. A massive object is a collective form of a huge number of atomic nuclei (i.e. proton and neutron) and a proton may be considered as a Gaussian wave packet [3]. The pressure distribution and energy density distribution inside a proton may be encoded by gravitational form factors of proton [4-6]. However, Gaussian Wave Function of a proton may be presented as given below [3]

$$\psi(r) = (2\pi R_P^2/3)^{-3/4} e^{-(3/4R_P^2)\,r^2}$$

where, R_P is rms radius of the localized proton probability distribution. If, we read $R_P = \sqrt{0.32}$ fm which is r.m.s mass radius of proton [6, 7] then, we obtain from the above equation

$\psi(r) = 1.35 \times e^{-\,2.34\,r^2}$ (1)

Now, position probability density $(\psi^*\psi)$ for a proton obtained from the above equation implies that at $r = 0$, position probability density of a Gaussian Wave Packet is maximum and that is zero at $r = \infty$. This picture gives a description that the particle is not confined in a fixed width rather, widespread from $r = 0$ to $r = \infty$ followed by the position probability density distribution. Now, one of the possible definitions of quantum energy density [8] is

$\rho(r) = \frac{\hbar^2}{2\mu}(\nabla\psi^*).(\nabla\psi) + \psi^* V(r)\psi$ (2)

For a static proton, total kinetic energy is zero and total potential energy is 0.938 GeV which is the rest mass energy of a proton. Therefore, with the help of (1), equation (2) leads to read $\rho(r) = 0.938 \times \psi^*\psi$ which is finally $\rho(r) = 1.7 \times e^{-\,4.68\,r^2}$ (3)

where, $\rho(0) = 1.7\ \text{GeV/fm}^3$ is the central mass density. Now, validity of equation (3) may be verified by the results as given below

A. $\rho(0) = 1.7\ \text{GeV/fm}^3$, the central mass density of a proton is estimated from the chiral quark soliton model [9]. It is the same as that in equation (3).

B. $\int_0^\infty 4\,\pi\, r^2\, \rho(r)\, dr = 0.935$. This result shows a good agreement with total rest mass of a proton.

C. The r.m.s. mass radius of a proton [7, 9], $R_P = \sqrt{0.32}$ fm makes the good agreement with the result obtained from relation [5] as given below

$$R_P = \sqrt{\frac{\int_0^\infty r^2 \rho(r) dv}{\int_0^\infty \rho(r) dv}} = \sqrt{\frac{\int_0^\infty 4\pi r^4 \rho(r) dr}{\int_0^\infty 4\pi r^2 \rho(r) dr}} = \sqrt{0.32} = 0.566$$

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Therefore, one may assume, the wave function of a proton and energy density distribution function inside a proton given in equation (1) and (3) respectively are the correct.

Next, we are interested to establish a function for radial pressure distribution inside a proton. Now, the data roughly collected from the diagram of radial pressure distribution inside the proton given in Ref [4] is as shown in the Table-1.

r (fm)	0.00	0.10	0.25	0.60	0.94	1.30
$r^2\ p(r)$ (GeV/fm)	0.0	3.6×10^{-2}	7.5×10^{-2}	0.0	-3.3×10^{-2}	-2.1×10^{-2}

Table-1: data on radial pressure distribution inside a proton, collected from Ref [4]. The distribution has a positive core and a negative tail of the $r^2\ p(r)$ distribution as a function of r, with a zero crossing near r = 0.6 fm. The maximum of the positive pressure is near r = 0.25 fm, and the maximum of the negative pressure is near r = 0.94 fm.

It is very interesting that data in the Table-1 make a very good agreement with the pressure distribution function as proposed below

$p(r) = 7.2(1 - \frac{r}{0.6})e^{-5r}$ (4)

where, pressure at the center of a proton is about $7.2\ \mathrm{GeV/fm^3}$ or, 11.5×10^{35} pascal. Now, exact values of $r^2\ p(r)$ obtained from equation (4) are shown in Table 2 which is almost the same as that in Table-1

r (fm)	0.0	0.10	0.25	0.60	0.94	1.30
$r^2\ p(r)$ (GeV/fm)	0.0	3.63×10^{-2}	7.52×10^{-2}	0.0	-3.28×10^{-2}	-2.13×10^{-2}

Table-2: exact data on radial pressure distribution inside a proton obtained from equation (4).

Also, the above function completely satisfies the von Laue stability condition [5]:

$$\int_0^{\infty} r^2 \mathrm{p}(r) dr = 0$$

where , $\int_0^{0.6} r^2\ p(r)\ dr = 25.8\,0$ MeV , $\int_{0.6}^{\infty} r^2\ p(r) dr = -25.8\,0$ MeV

Equation (4) leads to the Figure-1 and this diagram is almost the same as that in Ref [4]. These mean that equation (4) complies with data and diagram of pressure distribution given in Ref. [4]. Therefore, equation (4) may be considered as a possible radial pressure distribution function inside a proton.

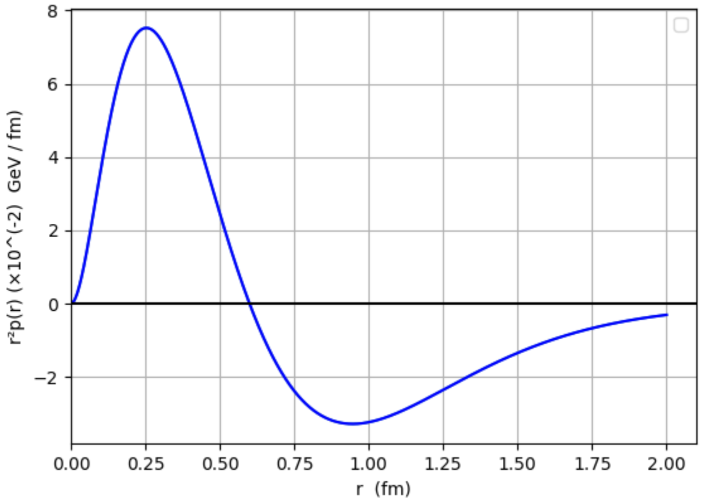

Figure-1 : Radial pressure distribution inside proton

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This work is a theoretical study but, somewhat it has experimental background. However, it is concluded that proton is a kind of Gaussian Wave Packet; inside this wave packet, energy density distribution and pressure distribution follow the relations given in equations (3) and (4) respectively. These relations may play an important role in formation of tress energy tensor inside a proton. So, it may help in study on the fundamental gravitational properties of protons.

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