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CREDIT RISK ASSESSMENT USING DEFAULT MODELS: A REVIEW

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ABSTRACT

Credit risk, also known as default risk, is the likelihood of a corporation losing money if a business partner defaults. If the liabilities are not met under the terms of the contract, the firm may default, resulting in the loss of the company. There is no clear way to distinguish between organizations that will default and those that will not prior to default. We can only make probabilistic estimations of the risk of default at best. There are two types of credit risk default models in this regard: structural and reduced form models. Structural models are used to calculate the likelihood of a company defaulting based on its assets and liabilities. If the market worth of a company's assets is less than the debt it owes, it will default. Reduced form models often assume an external cause of default, such as a Poisson jump process, which is driven by a stochastic process. They model default as a random event with no regard for the balance sheet of the company. This paper provides a Review of credit risk default models.

Keywords: *Credit Risk, Default Models, Structural Models, Reduced-form Models, Poisson jump process.*

INTRODUCTION

The uncertainty about a company's ability to service its debts and commitments is known as credit risk or default risk. It's the risk of a loss occurring as a result of a borrower's failure to repay a loan or meet contractual obligations. It refers to the likelihood of a corporation losing money if a business partner defaults. If the liabilities are not met under the terms of the contract, the firm may default, resulting in the loss of the company. Credit, commerce, and investment operations, as well as the payment system and trade settlement, all result in liabilities. Credit risk modelling is difficult due to the fact that company default is not a common occurrence and usually comes unexpectedly. However, when a creditor defaults, it frequently results in significant losses that cannot be predicted in advance, therefore effectively measuring and managing credit risk can reduce the severity of a loss (Mianková et al. 2014). Along with market risk and operational risk, credit risk is one of three key hazards that all financial markets must report and retain capital against. It shows the likelihood of the company losing money if a business partner fails. We can attribute this failure to a failure to meet contractual duties, which results in a company loss (Kollár 2014). According to Bielecki and Rutkowski (2004), Credit risk has three components: (i) default risk, which is the risk that the issuer or counterparty will fail to honour the terms of the obligation stated in a financial contract; (ii) spread risk, which is the risk of loss or underperformance of an issue or issues due to an increase in the credit spread; and (iii) downgrade risk, which is the risk of credit ratings deterioration.

DEFAULT MODELS

Default models employ market data to model the occurrence of a default event. They are created by financial organisations to estimate the likelihood of a corporate or sovereign entity defaulting on its credit obligations. These models have evolved into two distinct types of models: structural and reduced form models (Elizalde, 2005).

I. STRUCTURAL MODELS

This is the first group of default models that look at the structure of the company's capital and are based on its value. Merton (1974) pioneered structural models, which use the Black-Scholes option pricing framework to characterise default behaviour. They're used to figure just how likely a company is to default based on the value of its assets and liabilities. They make the assumption that they have complete knowledge of a company's assets and liabilities, leading in a predicted default time. These models indicate that default risks arise when the value of a company's assets falls below its outstanding debt at the maturity date (Saunders and Allen 2002). These models are designed to show a direct link between default risk and capital structure. The fundamental disadvantage of this strategy is that it ignores the market value of a company's assets and treats debt as an option on those assets, and the default event is predictable (Chatterjee, 2015).

II. REDUCED FORM MODELS

The reduced form models are the second type of default models. The relationship between default and firm value is not explicitly included in these models. Defaulting is viewed as an unanticipated occurrence that can be influenced by a variety of market circumstances. Reduced form models often assume an external cause of default, such as a Poisson jump process, which is driven by a stochastic process. They model default as a random event with no regard for the balance sheet of the company. A Poisson event is the term used to describe this type of random event. This technique to credit risk modelling is also known as default intensity modelling because Poisson models look at the arrival rate, or intensity, of a certain event. The probability or intensity of default, as well as the mean recovery rate, are calculated using reduced-form credit risk models, which employ the observed market credit spread (Acharya and Carpenter 2002). The Jarrow and Turnbull (1995) model, which uses multi-factor and dynamic analysis of interest rates to compute the probability of default, is one of the first reduced-form models.

LITERATURE REVIEW

I. Structural models

Merton (1974) calculated a company's credit risk by imagining its stock as a call option on its assets. Merton characterised a firm's asset value as a lognormal process and assumed that if the asset value fell below a specific default boundary, the firm would default. The default option was available only once, at maturity. The firm's equity was created via a call option on the underlying assets. The benefit of this model is that it can be used for any publicly listed company and that data from the stock market may be used instead of financial data. It can also be used to predict what will happen in the future. The use of this approach in everyday practise, on the other hand, highlighted some of its flaws. The model's credit spreads, which are premiums to risk-free interest rates, are typically lower than the real spreads. The assumptions of Merton's model scarcely resemble reality. Previous experience indicates that the company would struggle to pay its debts for a long time before the value of its assets falls below the value of its liabilities (Mianková 2015). Extensions to the Merton model can help to overcome some of the model's flaws. The most well-known and commonly utilised is Keaholfer, McQuown, and Vasicek (KMV), which was discovered in 1974 based on Merton's bond pricing model assumptions (Kliestik et al. 2015).

The Merton model is based on the idea of considering a company's equity as a call option on its assets, allowing Black-Scholes option pricing methods to be used. Assume a corporation has an asset A_t at time t financed by stock equity E_t and zero-coupon debt D_t with a face amount of K maturing at time $T > t$, and a capital structure determined by the balance sheet relationship:

$$A_t = E_t + D_t \tag{1}$$

The company defaults on its debt at T if $A_T < K$. In the case $A_T > K$ the company's debtholders can be paid the full amount K . Shareholders' equity value is given by $A_T - K$. Therefore, equity value at time T can be written as:

$$E_T = \max\left(A_T - K, 0\right) \tag{2}$$

The asset value A_T is assumed to follow a geometric Brownian motion (GBM) process, with risk-neutral dynamics given by the stochastic differential equation:

$$\frac{dA_t}{A_t} = rdt + \sigma_A dW_t \tag{3}$$

where W_t is a standard Brownian motion under risk-neutral measure, r denotes the continuously compounded risk-free interest rate, and σ_A is the asset's return volatility. Applying the Black-Scholes formula for European call option we obtain:

$$E_t = A_t N\left(d_1\right) - Ke^{-r(T-t)} N\left(d_2\right) \tag{4}$$

where $N(\cdot)$ denotes the $N(0,1)$ cumulative distribution function, with the quantities d_1 and d_2 given by:

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$$d_1 = \frac{\ln\left(\frac{A_t}{K}\right) + \left(r + \frac{1}{2}\sigma_A^2\right)(T-t)}{\sigma_A\sqrt{T-t}}, \quad (5)$$

$$d_2 = \frac{\ln\left(\frac{A_t}{K}\right) + \left(r - \frac{1}{2}\sigma_A^2\right)(T-t)}{\sigma_A\sqrt{T-t}}. \quad (6)$$

The continuously compounded credit spread s is given by:

$$s = -\frac{1}{T-t}\ln\left[N(d_2) - \frac{A_t}{K}e^{r(T-t)}N(-d_1)\right] \quad (7)$$

Equation (7) allows us to solve for credit spread when asset level and return volatility (A_t and σ_A) are available for given t, T, K, and r . To get A_t and σ_A we need to assume another geometric Brownian motion model for equity price E_t and applying Ito's Lemma to show that instantaneous volatilities satisfy:

$$A_t\sigma_A\frac{\partial E_t}{\partial A_t} = E_t\sigma_E. \quad (8)$$

Black-Scholes call option delta can then be substituted into (8) to obtain:

$$A_t\sigma_A N(d_1) = E_t\sigma_E \quad (9)$$

where equity price E_t and its return volatility σ_E are observed from equity market. Finally, (4) and (8) can be solved simultaneously for A_t and σ_A, which are used in (7) to determine credit spread s.

By modelling the evolution of firm value as a jump-diffusion process, Zhou (1997) created a new structural approach to estimating credit risk and evaluating default-risky instruments. A firm can default instantly due to a quick reduction in its value in a jump-diffusion process. They also naturally linked recovery rates to firm valuation upon default, resulting in endogenous variance in recovery rates in the model. The findings suggested that both the diffusion and jump processes could be key components of a structural debt valuation model. They proposed that their paper's valuation framework be expanded to include more institutional elements such as floating rate coupon payments and bond indenture provisions that may require a firm to return its lenders recovered values at default time if the bond defaults before maturity. He built a continuous-time valuation framework for hazardous debt by extending the Merton-Black-Cox-Longstaff-Schwartz approach and modelling the evolution of firm's value as a jump-diffusion process. He came up with the model by making a list of assumptions. Some of them are similar to Merton's (1974), Black and Cox's (1976), and Longstaff and Schwartz's (1977) works (1995).

1. Let V denote the total market value of the assets of the firm. The dynamics of V are given by the following jump-diffusion process:

$$dV/V = (\mu - \lambda v)dt + \sigma dZ_1 + (\Pi - 1)dY \quad (1)$$

where

- μ, v, λ and σ are positive constants
- Z_1 is a standard Brownian motion
- dY is a Poison process with intensity parameter λ
- $\Pi > 0$ is the jump amplitude with expected value equal to $v+1$, and
- dZ_1, dY and Π are mutually independent.

v equals the expected value of jump component $(\Pi - 1)$, μ represents the expected instantaneous rate of change of firm's value. They assume that Π is identically and independently distributed log-normal random variable, such that:

$$\ln(\Pi) \sim N(\mu_\pi + \sigma_\pi^2) \quad (2)$$

Implying that

$$v : E[\Pi - 1] = \exp(\mu_\pi + \sigma_\pi^2/2) - 1$$

The diffusion process (1) describes the natural variation in a firm's value that occurs as a result of gradual changes in economic conditions or the receipt of new knowledge, resulting in marginal changes in the firm's value. The jump component reflects the abrupt changes in a firm's value as a result of critical new information that has a significant impact on the firm's market value. A jump-diffusion process for a firm's value seems ideal for simulating a firm's default risk, given that a firm's value moves virtually continuously most of the time and that the market value of a firm can decrease drastically in the case of a default.

2. The capital asset pricing model (CAPM) holds for equilibrium security returns and the jump component of firm's value equation (1) is purely firm-specific and is uncorrelated with the market.

3. The Modigliani-Miller theorem that the value of the firm is invariant to its capital structure holds. The changes in capital structure, such as $\text{debt}/\text{equity}$ ratio and payments of coupons and principle, do not affect the firm's value V.

4. They assume perfect, frictionless markets in which securities trade in continuous time. Arbitrage opportunities do not exist.

5. There exists a positive threshold value K for the firm at which financial distress occurs. The firm continuous to operate and to be able to meet its contractual obligations as long as $V > K$. However, if its value V falls to or below the threshold level K, it defaults on all of its obligations immediately and some form of corporate restructuring takes place.

6. The firm issues both equity and debt (bonds). If it defaults during the life of a bond, the bond holder receives $1-w(X_s)$ times the face value of the security at maturity T. Here $s=\min(\tau,T)$ with τ being the time of default and $X := V/K$ is the ratio of the firm's value V to the threshold level K. w is usually a non-increasing function of X, that is the inequality $w'(X) \le 0$ holds. The factor w represents the percentage write-down on a bond if there is a reorganization of the firm. When $w=0$, there is no write-down and bondholders are not affected by the firm's reorganization. When $w=1$, bondholders receive nothing in a reorganization. In general, w differs across various bond issues in the firm's structure, it is a bond specific.

7. The short-term risk-free interest rate r is constant over time.

Assumption (1) and the definition that $X = V/K$ yield immediately:

$$dX/X = (\mu - \lambda v)dt + \sigma dZ_1 + (\Pi - 1)dY \qquad (3)$$

Let H be the price of any derivative security with payoff at time T contingent on the firm's X. Using Merton's (1976) result, the assumption that the jump risk is not systematic and that arbitrage opportunities are excluded, the derived price H must satisfy the following partial differential equation:

$$\frac{1}{2}\sigma^2 X^2 H_{XX} + (r - \lambda v) X H_X - rH + \lambda E_t\left[H(X\Pi, T) - H(X, T)\right] = H_T$$

(4)

Equation (4) does not depend on either the risk-aversion coefficient or the physical drift of the firm's X, as expected from standard no-arbitrage approach for pricing derivative securities. The value of any derivative security can be obtained by solving the equation subject to appropriate boundary conditions.

Crosbie and Bohn (2003), Kealhofer (2003), and Vasicek (1984) devised the MKMV technique, which uses the Vasicek-Kealhofer (VK) model to offer a term-structure of physical default risk probability. This model considers equity to be a perpetual down-and-out option on a company's fundamental assets. Short-term obligations, long-term liabilities, convertible debt, preferred equity, and common equity are among the five categories of liabilities that can be accommodated under this model. MKMV derives a firm's market value of assets and associated asset volatility using the option-pricing equations derived in the VK framework. Empirically, the default point term-structure is determined. MKMV creates a Distance-to-Default (DD) term-structure by combining market asset value, asset volatility, and the default point term-structure. Using an empirical mapping between DD and historical default data, this term structure is transformed to a physical default probability.

$$DD_T = \frac{\log\left[\frac{A}{X_T}\right] + \left(\mu - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} \quad (1)$$

The interpretation of X_T in the VK model differs slightly from that of the Merton model. If asset value A falls below X_T at any point in time during the analytical stage of the model, the firm is regarded to be in default. The VK model estimates a term-structure of this default barrier in the DD-to-EDF empirical mapping step, resulting in a DD term structure that may be transferred to a default-probability term-structure, hence the subscript T for default barrier X . According to Huang (2003), the default probability created by the MKMV implementation of the VK model is known as the Expected Default Frequency or EDF credit measure. After obtaining the EDF term structure, a cumulative EDF term structure up to any term T referenced to as $CEDF_T$ can be calculated. This is then converted to a risk-neutral cumulative default probability $CQDF_T$ using the following equation:

$$CQDF_T = N\left[N^{-1}\left[CEDF_T\right] + \lambda \cdot sqrt\left(R^2\right) \cdot sqrt\left(T\right)\right] \quad (2)$$

where R^2 is the square of correlation between the underlying asset returns and the market index returns, and λ is the market Sharpe ratio. The spread of a zero-coupon bond is obtained as:

$$s = -\frac{1}{T}\log\left[1 - LGD \cdot CQDF_T\right] \quad (3)$$

where LGD stands for the loss given default in a risk-neutral framework. The floating leg of a simple CDS (i.e. a single payment of LGD paid out at the end of the contract with a probability of $CQDF_T$) can also be approximated with this relationship.

Jarrow et al. (2003) conducted a robust test of Merton's structural credit risk model that did not rely on either estimated firm value parameters or projected default probabilities. They also used the Merton model to provide a test for the consistency of observed changes in debt and equity prices (positive or negative changes). The data significantly refuted Merton's structural model for all enterprises studied and for all debt difficulties considered. Only the monitoring of equity prices, debt prices, and the spot rate of interest was required for their testing approach. Over the period of February 6, 1992, to March 12, 2001, they tested the Merton model on five distinct companies' debt concerns in various sub-intervals, using both weekly and monthly observation intervals. However, because they did not look into the consequences of its extensions and generalizations, their testing methods may be used to look into these as well. They emphasized that Merton's model presupposes complete markets that are frictionless, competitive, and arbitrage-free, and that the firm's balance sheet can be stated as:

$$V_t = D_t\left(T, F\right) + E_t \quad (1)$$

where V_t is the value of the firm's assets at time t assumed to follow a diffusion process, D_t is the value of the firm's debt at time t, and E_t is the value of the firm's equity at time t. The debt is assumed to be a zero-coupon bond with maturity T and face value F. Let r_t denote the (default free) spot rate of interest at time t, assumed to be non-random. Using the call option analogy to the firm's equity, Merton (1974) develops a pricing formula for debt $D_t = D_t\left(V_t, r_t : T, F\right)$ that depends on the firm's value (and the parameters of its stochastic process, the volatility), the characteristics of the debt (face value and maturity date), and the spot rate of interest (r_t). Let $dV_t / V_t = \alpha dt + \sigma dW_t$ with W_t a standard Brownian motion. Then,

$$D_t = V_t N\left(h_1\right) + Fe^{-\int_t^T r_u du} N\left(h_2\right) \quad (2)$$

where

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$$h_1 = \frac{\ln\left(Fe^{-\int_t^T r_u du} / V_t \right) - \frac{1}{2}\sigma^2 (T-t)}{\sigma\sqrt{(T-t)}} \quad (3)$$

$$h_2 = -h_1 \sigma \sqrt{(T-t)} \quad (4)$$

$N(\cdot)$ is the cumulative standard normal distribution function.

Using Merton's model, Hull et al. (2004) investigated credit risk and volatility skews. They suggested a method for estimating the model's parameters using implied volatilities of stock options. They compared their implementation of Merton's model against the standard way to implementation using data from the credit default swap market. Their proposed Merton model implementation outperformed the approach's standard implementation.

Elizalde (2005) examined the structural approach to credit risk modelling, taking into account both the situation of a single firm and the scenario of firms with default dependencies. They looked at the Merton (1974) model and the first passage models (FPM) (Black and Cox 1976) in the single company situation, assessing its main properties and extensions. Finally, structural models with state-dependent cash flows or debt coupons were examined. They discovered that, according to Merton's model, a company fails if its assets are less than its outstanding debt at the time of debt servicing. Defaults in the FPM technique occur when a firm's asset value falls below a specified threshold. Unlike Merton's approach, default can happen at any time. FPM defines default as the first time a firm's asset value falls below a certain threshold, allowing default to occur at any time. SDMs presume that some of the characteristics affecting a firm's ability to generate cash flows or its funding costs are state dependent, with states representing the economic cycle or the firm's external rating. The empirical testing of FPM and structural models in general, on the other hand, has not been very fruitful. They agree that structural bond pricing models do not effectively price corporate bonds, based on estimations from implementations, because they present the predictability of defaults and recovery rates, which is not true in real market conditions.

In the Black-Scholes-Merton paradigm, Kulkarni et al. (2005) modelled default probabilities and credit spreads for selected Indian enterprises. Over the sample period, they found that the objective probability estimates are greater than the risk-neutral estimates. The model's output performed well when compared to the Altman Z-score measure. The model, however, did not produce spreads as wide as those seen in the corporate bond market.

Tarashev (2005) compared the probability of default (PDs) generated by six structural credit risk models to ex post default rates to assess their performance experimentally. The paper uses firm-level data, in contrast to other studies seeking similar goals, and demonstrates that theory-based PDs tend to closely match the actual level of credit risk and explain for its time path. Simultaneously, non-modeled macro variables from the financial and real sides of the economy aid in significantly improving default rate estimates. he findings show that theory-based PDs do not adequately reflect credit risk's reliance on business and credit cycles. The majority of the optimistic conclusions about PD performance are attributed to models with endogenous default. Exogenous default frameworks, on the other hand, have a tendency to underestimate credit risk.

Using term structure of credit default swap (CDS) spreads and stock volatility from high-frequency return data, Huang and Zhou (2008) investigated specification analysis of structural credit risk models. Based on the simultaneous behavior of time-series asset dynamics and cross-sectional pricing errors, they provide consistent econometric estimate of pricing model parameters and specification tests. The conventional Merton (1974) model, the Black and Cox (1976) barrier model, and the Longstaff and Schwartz (1995) model with stochastic interest rates are all significantly rejected by their empirical testing. The double exponential jump-diffusion barrier model (Huang and Huang, 2003) outperforms the other two models significantly. The stationary leverage model of Collin-Dufresne and Goldstein (2001), which we cannot reject in more than half of our sample firms, is the best of the five models studied. However, empirical data show that conventional structural models, particularly for investment grade names, are unable to represent the dynamic behavior of CDS spreads and equity volatility. Given that equity volatility in structural models is time-varying, this finding provides direct evidence that using a structural model with stochastic asset volatility (as in Huang and Huang, 2003; Huang, 2005; Zhang, Zhou, and Zhu, 2009) can significantly improve model performance, particularly for investment-grade names. This suggests that time-varying asset volatility, which is not included in typical structural models, could play a role. Although these five models have different economic assumptions, they can all be integrated in the same underlying structure that comprises asset process, default boundary, and recovery rate requirements for the underlying company.

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Let V be the firm's asset process, K the default boundary, and r the default-free interest rate process. Assume that, under a risk-neutral measure,

$$\frac{dV_t}{V_{t-}} = (r_t - \delta)dt + \sigma_v dW_t^Q + d\left[\sum_{i=1}^{N_t^Q}\left(Z_i^Q - 1\right)\right] - \lambda^Q \xi^Q dt \qquad (1)$$

$$d\ln K_t = Kl\left[-V - \phi(r_t - \theta) - \ln(K_t/V_t)\right]dt \qquad (2)$$

$$dr_t = (\alpha - \beta r_t)dt + \sigma_r dZ_t^Q \qquad (3)$$

where $\delta, \sigma_v, Kl, V, \phi, \alpha, \beta, \sigma_r$ and $\theta = \alpha/\beta$ are constants, and W^Q and Z^Q are both one- dimensional standard Brownian motion under the risk-neutral measure and are assumed to have a constant correlation coefficient of ρ. In Eq. (1), the process N^Q is a Poisson process with a constant intensity $\lambda^Q > 0$, the Z_i^Q 's are $i.i.d$ random variables, and $Y^Q \equiv \ln\left(Z_1^Q\right)$ has a double-exponential distribution with a density given by:

$$f_{Y^Q}(y) = p_u^Q \eta_u^Q e^{-\eta_u^Q y} 1_{\{y \ge 0\}} + p_d^Q \eta_d^Q e^{-\eta_d^Q y} 1_{\{y<0\}} \qquad (4)$$

In equation (4), parameters $\eta_u^Q, \eta_d^Q > 0$ and $p_u^Q, p_d^Q \ge 0$ are all constants, with $p_u^Q + p_d^Q = 1$.The mean percentage jump size ξ^Q is given by:

$$\xi^Q = E^Q\left[e^{Y^Q} - 1\right] = \frac{p_u^Q \eta_u^Q}{\eta_u^Q - 1} + \frac{p_d^Q \eta_d^Q}{\eta_d^Q + 1} - 1 \qquad (5)$$

All five models considered in this analysis are special cases of the general specification in Eqs. (1) - (3). For instance, if the jump intensity is zero, then the asset process is a geometric Brownian motion. If both β and σ_r are zero, then the interest rate is constant, an assumption made in the three one-factor models. They assume a constant recovery rate for comparison with other studies and because the CDS database that they used includes the recovery rate estimates. Under each of the five structural models, it is straightforward to calculate the CDS spread. Let $Q(0,T)$ denote the survival probability over $(0,T]$ under the T -forward measure. Then the CDS spread of a T-year CDS contract is given by:

$$cds(0,T) = \frac{(1-R)E^Q\left[\exp\left(-\int_0^T r(u)du\right)I_{\{\tau<T\}}\right]}{\sum_{i=1}^{4T} D(0,T_i)Q(0,T_i)/4} \qquad (6)$$

where R is the recovery, r is the interest rate process, $D(0,\cdot)$ the default-free discount function, τ the default time, and $I_{\{\}}$ the indicator function, and $E^Q[\cdot]$ the expectation under the risk-neutral measure. To simplify the computation, they followed the literature to make the standard assumption that the settlement of the contract occurs on the next payment day. It then follows from Eq. (6) that:

$$cds(0,T) = \frac{(1-R)\sum_{i=1}^{4T} D(0,T_i)\left[Q(0,T_{i-1}) - Q(0,T_i)\right]}{\sum_{i=1}^{4T} D(0,T_i)Q(0,T_i)/4} \qquad (7)$$

As a result, the implementation of a structural model amounts to the calculation of the survival probability $Q(0,\cdot)$. In the Merton (1974) and the Black and Cox (1976) models, $Q(0,\cdot)$ has closed form solutions. The survival probability in the double exponential jump diffusion model and the two-factor models do not have a known closed form solution but can be easily calculated using a numerical method.

Schaefer and Strebulaev (2008) investigated the impact of structural credit risk models on corporate bond hedging ratios. They demonstrated that, whereas structural credit risk models are poor predictors of bond prices, they are highly accurate predictors of corporate bond returns' sensitivity to changes in the value of stock (hedge ratios). This is significant because it implies that structural models' poor performance may be due to the influence of non-credit factors rather than a failure to reflect the credit exposure of corporate debt. The key finding of their study is that even the most basic structural model for corporate debt pricing developed by Merton (1974): the risk structure of interest rates, produces hedging ratios that are not rejected in time-series tests. They discover, however, that the Merton model, with or without stochastic interest rates, fails to explain why corporate bonds' interest rate sensitivity is so low.

Wang (2009) gave an overview of a regularly used structural credit risk modelling approach that the actuarial community is less aware with. He emphasized that the Merton model is based on the idea of treating a company's equity as a call option on its assets, allowing Black-Scholes option pricing methods to be used. Credit spread, according to Merton's model, compensates for credit risk, which is linked to structural determinants (assets, liabilities, etc.). However, empirical evidence suggests that the Merton model undervalues credit spreads, especially short-term spreads for high-quality debtors. This flaw has lately been addressed by a number of expanded models, including studies by Black and Cox (1976).

Existing structural models, according to Davydenko (2012), negate liquidity issues as the primary predictors of default for some enterprises, notably those with significant external funding costs. He emphasized that if external financing are too expensive, a liquidity crisis may require rearrangement, even though the going-concern surplus remains significant. According to empirical evidence, structural models need to be theoretically extended to include the risk of enterprises defaulting due to liquidity shortages and high funding costs.

The Merton model was used by Valáková and Klietik (2014) to analyze credit risk. The possibility of debtor default or the difference between the value of the company assets and the default barrier expressed as a number of standard deviations is how the model depicts credit risk. They discovered that default happens in the Merton model when the market value of the company's assets is less than the book value of its obligations. All relevant information about the company's risk profile is included in accounting and market pricing of securities issued by the company, which is a major prerequisite of the technique based on market data analysis.

Hoang and Vuong (2015) used the Merton model to construct a framework for measuring credit risk with jumps. They studied a Merton model for default risk, in which a Brownian motion and a compound Poisson process drive the firm's value. The findings revealed that the firm's worth can fluctuate at random, not only in a continuous but also cumulatively discrete manner.

II. Reduced form models

Chen and Panjer (2002) used a jump-diffusion process to represent firm value evolution, where the instantaneous percentage of change in firm value is made up of the change from a systematic diffusion process plus the change from a nonsystematic leap. They presume that the credit spread must be consistent with the market spread, unlike typical structural models that have distinct credit spreads from market spreads. Because the diffusion process evolves at a riskless rate, their research determines the implied jump distribution using the market credit spread. According to their findings, their proposed model establishes an intensity process, allowing a structural model and a reduced-form model to be merged. Their research developed an exponential-lognormal jump-diffusion process, which produced a distributed recovery that is consistent with the market experience.

Hull and White (2000) developed a methodology for valuing credit default swaps when the payoff is contingent on default by a single reference entity and there is no counterparty default risk. Instead of using a hazard rate for the default probability, this model incorporates a default density concept, which is the unconditional cumulative default probability within one period no matter what happens in other periods. By assuming an expected recovery rate, the model generates default densities recursively based on a set of zero-coupon corporate bond prices and a set of zero-coupon Treasury bond prices. Then the default density term-structure is used to calculate the premium of a credit default swap contract. The two sets of zero-coupon bond prices can be bootstrapped from corporate coupon bond prices and treasury coupon bond prices. They show the credit default swap (CDS) spread s to be:

$$s = \frac{\int_0^T \left[1 - R\left(1 + A(t)\right)\right] q(t) v(t)\, dt}{\int_0^T q(t)\left[u(t) + e(t)\right] dt + \pi u(t)} \tag{1}$$

where:

T : the life of the CDS contract.

$q(t)$: the risk-neutral default probability density at time t.

$A(t)$: the ac crued interest on the reference obligation at time t as a percent of face value.

π : the risk-neutral probability of no credit event over the life of the CDS contract.

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Peer-Reviewed, Multidisciplinary & Multilingual Journal
ISSN: 2321-1520 E-ISSN: 2583-3537

w : the total payments per year made by the protection buyer.

$e(t)$: the present value of the accrued payment from previous payment date to current date.

$u(t)$: the present value of the payments at time t at rate of $1 on the payment dates.

R : the expected recovery rate on the reference obligation in a risk-neutral world.

The risk-neutral default probability density is obtained from the bond data using the relationship:

$$q_j = \frac{G_j - B_j - \sum_{i=1}^{j-1} q_i \alpha_{ij}}{\alpha_{jj}} \qquad (2)$$

where α_{ij} is the present value of the loss on a defaultable bond j relative to an equivalent default-free bond at time t_i. α_{ij} can be described as:

$$\alpha_{ij} = v(t_i)\left[F_j(t_i) - R_j(t_i)C_j(t_i)\right] \qquad (3)$$

C_j is the claim made on the jth bond in the event of default at time t_i, while R_j is the recovery rate on that claim. F_j is the risk-free value of an equivalent default-free bond at time t_i, while $v(t_i)$ is the present value of a sure payment of $1 at time t_i. Under this framework, one can infer a risk-neutral default risk density from a cross-section of bonds with various maturities. As long as the bonds measure the inherent credit risk and have the same recovery as used in the CDS, one should be able to recover a fair price for the CDS based on the prices of the obligors traded bonds.

A specification analysis of reduced-form credit risk models was undertaken by Berndt (2007). They evaluated numerous one-factor reduced-form credit risk models for actual default intensities using non-parametric specification tests devised by Hong and Li (2005). They used Moody's KMV estimations for actual default probabilities for 106 U.S. enterprises in seven industrial groups from 1994 to 2005 (Bohn et al. 2005 and Crosbie and Bohn 2003). Popular univariate affine model specifications were sharply rejected by the results. They hypothesized that the logarithm of the real default intensity follows an Ornstein-Uhlenbeck process, known as the Black-Karasinski (BK) model, for goodness-of-fit and model simplicity (1991). They discovered significant mean-reversion in actual log-default intensities for the BK model specification, with an average half-time of around 18 months. Findings also indicated the level of pairwise correlation in log-default intensities differed across industries.

The reduced form modelling approach for credit risk was provided by Jeanblanc and Lecam (2008) in a unified setting. A credit event was depicted as an inaccessible random time. They proposed two techniques to modelling default: the intensity-based approach, which is efficient when working with Cox process construction since the default time is generated from the intensity, and the hazard process approach, which gives the default time. The results demonstrated that knowing the intensity is not required to price contingent claims using the first method. The last method is particularly suited to studying models with incomplete observations, which is a method of studying a model where the default has an economic value (as in structural models) and when the default is unpredictable, giving nice spreads.

In a reduced form model of default spreads with Markov-switching macroeconomic factors, Dionne et al. (2011) investigated the ability of observed macroeconomic indicators and the probability of regime shifts to explain the proportion of yield spreads induced by the risk of default. They applied Bansal and Zhou's (2002) Markov-switching risk-free term structure model to corporate bonds, developing recursive formulas for default probability, risk-free and risky zero-coupon bond yields, and credit default swap premia. They used consumption, inflation, risk-free returns, and default data for Aa, A, and Baa bonds from 1987 to 2008 to calibrate their model. Macroeconomic factors were associated to two out of three dramatic spikes in default spreads during this sample period, according to the researchers. Both inflation and consumption growth were adversely associated to default spreads during these recessions, demonstrating that spread variations can be linked to macroeconomic undiversifiable risk. They also mentioned that the bond market's illiquidity is most likely the key reason for the discrepancy in default and credit spreads. They proposed two additions to their research: (i) taking into account alternative macro factors that are more closely linked to economic recessions than consumption, and (ii) explicitly including liquidity risk in the model.

VIDYA - A JOURNAL OF GUJARAT UNIVERSITY
Peer-Reviewed, Multidisciplinary & Multilingual Journal
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Liang and Wang (2012) proposed a reduced form credit risk model in which common shocks with regime-switching describe the default dependence structures among default intensity processes. They also came up with some closed-form formulations for the joint distribution of default timings and the basket default swap pricing formulas. They used correlated relations to depict the default dependence structure among the default intensity processes in their model.

Su and Wang (2012) used a reduced form model to analyze the valuation of European options with credit risk. They assumed that the interest rate follows the Vasicek model, and that the default intensity is determined by a jump diffusion process. They were able to establish the closed form formula for the option price. They then calculated the value of the vulnerable European call option by altering the probability measures. Finally, they calculated the effects of the recovery rate, correlation coefficients, and Poisson intensity on the option price using numerical analysis.

III. Comparison of Structural models and Reduced form models

Using a jump-diffusion technique, Chen and Panjer (2003) combined discrete structural models and reduced-form models in credit risk. They calculated the default probability and mean recovery rate by combining the market spread and the firm's capital structure. They used the credit spread to find the implied jump distribution. They demonstrated that credit spreads in the structural and reduced-form models are equivalent. The default probability and mean recovery rate were calculated using the market spread in the reduced-form model, whereas the default probability and mean recovery rate were calculated using the capital structure in the structural model. The degree of freedom is raised by adding a jump process to the diffusion process, but it is lowered by the market spread, according to the results. The credit spread difference between the structural model and the reduced-form model was eliminated by using the market spread to calculate the implied jump distribution, and the structural model and the reduced-form model may now be unified when the default can only occur at maturity. However, there is no mechanism for determining the jump distribution. The default probability (or intensity of default) and mean recovery rate are calculated from the market spread using model-specific assumptions in a reduced-form model, although the capital structure that triggers the default is rarely used.

From an information-based perspective, Jarrow and Protter (2004) evaluated structural vs reduced form credit risk models. Structural models presume that the modeler has access to the same data set as the business manager, which includes a thorough understanding of the firm's assets and liabilities. In the vast majority of cases, this knowledge results in a predictable default time. Reduced form models, on the other hand, presume that the modeler has the same set of data as the market's inadequate understanding of the firm's status. In the majority of circumstances, this incomplete knowledge results in an unavailable default time, as a result, they claim that the main contrast between structural and reduced form models is whether the information set is observed by the market or not, not whether the default time is predictable or inaccessible. They recommended reduced form models as the preferred methodology for pricing and hedging because they were built specifically to be based on the information available to the market.

Two structural models of credit risk (basic Merton and Vasicek-Kealhofer (VK)) and one reduced-form model (Hull and White 2003) were empirically compared by Arora et al. (2005). Default discrimination and relative value analysis are two useful reasons for credit models, according to them. They looked at how well the Merton and VK models could distinguish defaulters from non-defaulters using default probability derived from equities market data. They examined the HW model's ability to distinguish defaulters from non-defaulters using default probability derived from bond market data. They discovered that the VK and HW models outperform the simple Merton model on both the complete sample and relevant sub-samples, with comparable accuracy ratios. They also assessed each model's ability to anticipate spreads in the credit default swap (CDS) market as a measure of its relative value analysis capability. Except in circumstances where an issuer has a large number of bonds in the market, they discovered that the VK model performs better throughout the entire sample and relative sub-samples. In this scenario, the HW model is the most effective. On the structural side, a simple Merton model proved insufficient; proper framework adjustments were required to create the difference. The quality and quantity of data made a difference on the reduced-form side.

Jarrow (2009) examined the structural and reduced form credit risk models utilized in financial economics. They claimed that reduced form models, rather than structural models, should be used to price and hedge credit-risky instruments because structural models are static and do not represent the dynamic structure of credit risk. The sophistication of default contagion models, as well as the estimating processes used, must be increased to avoid a repeat performance. This enhancement will necessitate extensive study to identify dynamic models that represent default contagion while having parameters that can be estimated and values that can be computed. These include

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stochastic recovery rate models, which are needed to better represent the dynamic nature of default losses and the reliance of credit and liquidity risk, which is crucial for valuation, hedging, and capital determination.

Jarrow (2011) explored the theory and evidence of credit market equilibrium. The structural and reduced form models were reviewed as alternative paradigms for estimating credit risk. Their discussion was based on their understanding of credit market equilibrium. They demonstrated that in the borrowing and lending relationship, credit markets include asymmetric information, which influences equilibrium prices. Asymmetric equilibrium models are consistent with reduced form models, while structural models are not. Structural models should not be used for pricing, hedging, or risk management, as a result.

Mianková et al. (2015) calculated credit risk using reduced form models based on credit spreads and calculated the chance of default. Reduced-form models employ credit spreads as an input for calculating the probability of default, as opposed to structural models, which try to explain credit spreads through structural characteristics. They calibrated their model using observable market data, which is one of the most appealing aspects of this technique, and this flexibility is a significant benefit of these models. The results reveal that in these models, the structural characteristics of the company are insufficient to explain credit default swap (CDS) spreads, and empirical evidence shows that systematic risk has a significant impact on credit default swap prices (CDS). CDS pricing, on the other hand, which are derived using structural models, frequently overreact to increasing market volatility.

Table 1. Evolution of credit risk models

Main models	Related empirical studies	Treatment of default event
Structural models	Merton(1974), Merton(1976), Zhou (1997), Bohn et al. (2005), Jarrow et al. (2003), Hull et al. (2004), Elzade (2005), Black and Cox(1976), Kulkami et al. (2005), Tarashev (2005), Huang (2005), Zhang, Zhou and Zhu (2006), Huang and Zhou (2008), Huang and Huang (2003), Schaefer and Strebulaev (2008),Wang (2009), Davydenko (2012), Valášková and Klieštik (2014), Hoang and Vuong (2015), Longstaff and Schwartz (1995), Mišanková (2015), Saunders and Allen 2002, (Chatterjee, 2015)	-Aim to provide an explicit relationship between default risk and capital structure -Based on the value of the company -Examine the structure of the capital of the company -Endogenously specify default event and recovery rates -Calculate the probability of default of a firm based on the value of assets and liabilities -Assume complete knowledge of a company's assets and liabilities resulting in a predictable default event and recovery rates -Assume default risk occur at the maturity date if at that stage, the value of a company's assets fall below a debt threshold -Don't observe the market value of a firm's assets -Consider company's debt as an option on company's assets -Model a firm's asset value as a lognormal process
Reduced-form models	Acharya and Carpenter (2002),Jarrow and Turnbull (1995), Chen and Panjer (2002),	-They don't consider the relation between default and firm's value in an explicit manner -Treat default as unexpected event -They assume an exogenous cause of default driven by a stochastic process (Poison jump process) -They model default as a random event without any focus on the firm's balance sheet

	Berndt (2007), Hong and Li (2005), Jeanblanc and Lecam (2008), Dionne et al. (2011), Crosbie and Bohn (2003),Black and Karasinski (1991), Bansal and Zhou (2002), Liang and Wang (2012), Su and Wang (2012),	-Describe a random event of default as a Poison event (default intensity modeling) -Use market observed market credit spread to obtain the probability (intensity) of default and recovery rates -These models lack economic insights about occurrence of defaults
Comparison of Structural models and Reduced form models	Arora et al. (2005), Hull and White (2002), Jarrow (2009), Jarrow (2011), Mišanková et al. (2015)	Because structural models are static in character and do not represent the dynamic structure of credit risk, the literature argues that reduced form models, not structural models, are ideal for the pricing and hedging of credit-risky instruments. Structural models, with minor adjustments, can be effective in the pricing and hedging of credit securities.

CONCLUSION AND SUGGESTION FOR FUTURE RESEARCH

We have evaluated studies on default models' approach to credit risk in this paper. We looked at research on structural models, reduced form models, and comparisons between the two methods. Default models, in general, focus on the modelling of default events using market data. These models have evolved through time into two distinct types: structural and reduced form models. Credit risk is linked to underlying structural factors in structural models, which is a very appealing aspect. They allow for the use of option pricing methodologies and provide both an understandable economic interpretation and an endogenous explanation of credit defaults. As a result, structural models can help with not just asset value but also financial structure selection. The fundamental disadvantage of structural models is their complexity in implementation and the predictability of defaults and recovery rates, which is contradictory to market reality. Reduced form models calculate the likelihood (or intensity) of default as well as the mean recovery rate using the observed market credit spread. They specify recovery rates exogenously. These models suffer from a lack of economic insights into default occurrence; yet, they provide more functional form selection freedom. The analytical tractability, as well as the ease of implementation and calibration, are aided by this flexibility (compared to structural models). Reduced form models, on the other hand, may have high in-sample fitting features but limited out-of-sample prediction power due to their reliance on past data. Because structural models are static in nature and do not represent the dynamic structure of credit risk, empirical data suggests that reduced form models, not structural models, are ideal for the pricing and hedging of credit-risky instruments. It is necessary to make improvements to the estimating processes employed in structural models. This enhancement will necessitate extensive study to identify dynamic models that represent default contagion while having parameters that can be estimated and values that can be computed. These include stochastic recovery rate models to better represent the dynamic nature of default losses, as well as credit risk reliance and the incorporation of liquidity risk for valuation, hedging, and capital determination.

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